Web Interoperabitlity for Ontology Engineering with crowd 2.0

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1 Preliminaries

The analysis of the CDML common primitives supported by KF (see Table 1) led the definition of a fragment of each CDML to be considered in the followings theorems, which will be denoted by UML^{zero}, EER^{zero} and $ORM2^{zero}$, respectively, following a notation similar to the one defined in [1].

Definition 1. *Let $KF^{zero} = \{$Object type, role, binary relationship, object type subsumption, relationship subsumption, completeness constraint subsumption, disjoint object type subsumption, mandatory, object type cardinality constraint$\}$ the fragment of KF that includes all common primitives. We denote KF^{zero} conceptual schema by Σ_{KF}^{zero}.*

UML^{zero}, EER^{zero} and $ORM2^{zero}$ are defined considering the KF^{zero} corresponding primitive in UML, ER and ORM2, respectively (see Table 1).

We denote the UML^{zero}, EER^{zero} and $ORM2^{zero}$ conceptual schema by D_{UML}^{zero}, D_{EER}^{zero} and D_{ORM2}^{zero}, respectively.

Table 1: Metamodel primitives and their corresponding ones in each CDML. (*) binary relationships. (**) Only applied on more than two object types in a subsumption. (***) it represented differently by means of ORM 2 Value types. (****) it not included in ER diagrams.

KF	UML	ER/EER	ORM 2
`Object type`	Class	Entity	Entity type/Object type
`Role`	Association End	Component of a relationship	Role
`Relationship` (*)	Association	Relationship	Fact type
`Object type subsumption`	Subclass	Subtype	Subtype
`Relationship subsumption`	Subtyping of associations	Subtyping of relationships	Subset constraint on fact type
`Completeness constraint` (**)	Complete	Total	Total
`Disjoint object type` (**)	Disjoint	Disjoint	Exclusive
`Mandatory`	Mandatory role	Mandatory	Mandatory
`Object type cardinality constraint`	Multiplicity constraint	Cardinality constraint	Frequency constraint
`Attribute` (***)	Attribute	Attribute	**Absent**
`Data type` (****)	Literal	**Absent**	Data type

1.1 Embedding Rules

KF/DL Embedding Rules

Table 2: KF/DL Embedding Rules

KF	**DL**
`Object type O`	Concept O
`Role` $r_{endConcept}$	Role $r_{endConcept}$
`Data Type D`	Concept D
`Attribute A of data type DT for the object type O`	Role a $\exists a \sqsubseteq O$ $\exists a^{-} \sqsubseteq DT$ $O \sqsubseteq \exists a \sqcap (\leq 1\ a)$
`Binary Relationship R between O1 and O2`	Concept R $\exists r_{o1} \sqsubseteq R$ $\exists r_{o1}^{-} \sqsubseteq O1$ $\exists r_{o2} \sqsubseteq R$ $\exists r_{o2}^{-} \sqsubseteq O2$ $R \sqsubseteq \exists r_{o1} \sqcap (\leq 1\ r_{o1}) \sqcap \exists r_{o2} \sqcap (\leq 1\ r_{o2})$
`Object type O cardinality constraint:` (1)Range `(min, max)`	 $O \sqsubseteq (\geq \min\ r_o^{-}) \sqcap (\leq \max\ r_o^{-})$
(2)Range `(.. max)`	$O \sqsubseteq (\leq \max\ r_o^{-})$
(3)Range`(min ..)`	$O \sqsubseteq (\geq \min\ r_o^{-})$
`Mandatory role` r_o	$O \sqsubseteq \geq 1\ r_o^{-}$
`Object type subsumption`	$OSub \sqsubseteq OSup$
`Disjoint object type subsumption`	$O_1 \sqsubseteq OSup$ $O_2 \sqsubseteq OSup$ $\vdots$ $O_n \sqsubseteq OSup$ $O_i \sqsubseteq \prod_{j=i+1}^{n} \neg O_j$, for $i = 1, \ldots, n-1$
`Completeness object type subsumption`	$O_1 \sqsubseteq OSup$ $O_2 \sqsubseteq OSup$ $\ldots$ $O_n \sqsubseteq OSup$ $OSup \sqsubseteq O_1 \sqcup O_2 \sqcup \ldots \sqcup O_n$
`Relationship Subsumption`	$RChild \sqsubseteq RParent$

2 UML

Definition 2. *Let UML^{zero} be the fragment of UML that includes the following primitives:*

- *Classes*
- *Association end*
- *Binary association with multiplicity constraints*
- *Classes subsumption*
- *Relationship subsumption*
- *Mandatory*
- *Classes completeness constraints Subsumption*
- *Classes disjoint subsumption*

We define D^{zero}_{UML} as UML^{zero} conceptual model.

Theorem 1. *Let D^{zero}_{UML} be an UML^{zero} conceptual schema, Σ^{zero} be the corresponding KF conceptual schema built using the interoperabilities rules and $\Sigma^{zero}_{\mathcal{ALCIN}}$ the $\mathcal{ALCIN}$ knowledge base constructed as described in Table 2.*

A class C is consistent in D^{zero}_{UML} if and only if the corresponding concept encoding of C, is satisfiable in $\Sigma^{zero}_{\mathcal{ALCIN}}$.

Proof. We assume that the signatures of symbols representing classes, relationships and roles are disjoint. In the scope of this proof we will consider the FOL assertions in [2] as the semantic of a D^{zero}_{UML} conceptual schema.

($\Rightarrow$) Let $\mathcal{I} = (\Delta^{\mathcal{I}}, .^{\mathcal{I}})$ be an instantiation of D^{zero}_{UML}, ie. a model of the corresponding FOL assertions, such that $C^{\mathcal{I}} \neq \varnothing$. Then we can build a model $\mathcal{J} = (\Delta^{\mathcal{J}}, .^{\mathcal{J}})$ of $\Sigma^{zero}_{\mathcal{ALCIN}}$ such that $C^{\mathcal{J}} \neq \varnothing$ as follows:

$\Delta^{\mathcal{J}} = \Delta^{\mathcal{I}} \cup \bigcup_{A \in \mathcal{A}} \{t_{(d_{C1}, d_{C2})} | (d_{C1}, d_{C2}) \in A^{\mathcal{I}}\}$ where $\mathcal{A}$ denotes the set of all binary association

$C^{\mathcal{J}} = C^{\mathcal{I}}$ for each concept D correponding to classes in D^{zero}_{UML}

Finally, for each agregation and binary association without association class, A, we define

$$A^{\mathcal{J}} = \{t_{(d_{C1}, d_{C2})} | (d_{C1}, d_{C2}) \in A^{\mathcal{I}}\}$$

and for each $\mathcal{ALCIN}$ role modeling the component of the association A associated with the concept Ci, we define

$$a^{\mathcal{J}}_{C1} = \{(t_{(d_{C1}, d_{C2})}, d_1) | (d_{C1}, d_{C2}) \in A^{\mathcal{I}} \ \wedge \ d_{C1} \in C1^{\mathcal{I}}\}$$

and

$$a^{\mathcal{J}}_{C2} = \{(t_{(d_{C1}, d_{C2})}, d_2) | (d_{C1}, d_{C2}) \in A^{\mathcal{I}} \ \wedge \ d_{C2} \in C^{\mathcal{I}}_2\}$$

Trivially, $C^{\mathcal{J}} = C^{\mathcal{I}} \neq \varnothing$. As for the rest of the expressions in Σ^{zero}, it must be verified that for all $\mathcal{I}$ that are model of Σ^{zero}_{UML}, there is a $\mathcal{J}$ that is a model of the corresponding $\mathcal{ALCIN}$ knowledge base.

Binary relationships If a binary association A between two clases C1 and C2, with cardinality constraint card1-min and card1-max and card2-min and card2-max, respectively then the KF encoding built a binary relationship between two object type C1 and C2 with two roles a_{C1} and a_{C2}, which inherit cardinality constraints (KF Rules: **UML-A1**, **UML-R1**,**UML-MC1**). This encoding is then translate to DL as follow:

$$\exists a_{C1}.\top \sqsubseteq A$$
$$\exists a_{C1}^{-}.\top \sqsubseteq C1$$
$$\exists a_{C2}.\top \sqsubseteq A$$
$$\exists a_{C2}^{-}.\top \sqsubseteq C2$$

$$A \sqsubseteq \ \exists a_{C1} \sqcap (\leq 1 \ a_{C1}) \sqcap \exists a_{C2} \sqcap (\leq 1 \ a_{C2})$$

$$C1 \sqsubseteq \ (\geq card1 - min \ a_{C1}^{-}) \sqcap (\leq card1 - max \ a_{C1}^{-})$$
$$C2 \sqsubseteq \ (\geq card2 - min \ a_{C2}^{-}) \sqcap (\leq card2 - max \ a_{C2}^{-})$$

As $\mathcal{I}$ is a model then the following formulas are true:

$$\forall x, y.A(x, y) \supset C1(x) \wedge C2(y)$$
$$\forall x.C1(x) \supset (card1 - min \leq \#\{y|A(x, y)\} \leq card1 - max)$$
$$\forall y.C2(y) \supset (card2 - min \leq \#\{x|A(x, y)\} \leq card2 - max)$$

By definition,

$$A^{\mathcal{J}} = \{t_{(d_{C1},d_{C2})}|(d_{C1}, d_{C2}) \in A^{\mathcal{I}}\}$$
$$a_{C1}^{\mathcal{J}} = \{(t_{(d_{C1},d_{C2})}, d_1)|(d_{C1}, d_{C2}) \in A^{\mathcal{I}}\}$$
$$a_{C2}^{\mathcal{J}} = \{(t_{(d_{C1},d_{C2})}, d_2)|(d_{C1}, d_{C2}) \in A^{\mathcal{I}}\}$$

Therefore, $\mathcal{J}$ is a model for the first four $\mathcal{ALCIN}$ formulas, because they express the domain and range of roles a_{C1}, a_{C2} defined.

The fifth DL formula express that there is just one pair (d_{C1}, d_{C2}) in the A class and that this pair is in both roles, which is true because of the definition of $\mathcal{J}$.

Finally, the DL cardinality constraints formulas are satisfiables under the model $\mathcal{J}$ since every d_{C1} such that $C1(d_{C1})$ must keep to the cardinality of the elements $d_{C2} \in C2$ such that $A(d_{C1}, d_{C2})$. The claim holds for $C2(d_{C2})$.

Mandatory role This case can be considered as a binary relationship A in which one (or both) of the roles is mandatory. Considering that a_{C1} is a mandatory role of A associated with the concept $C1$, the following FOL formula

$$\forall x C1(x) \supset \exists y A(x, y)$$

The KF encoding is built from the rule (**UML-M1**) and the DL encoding is the same as a binary relationship, except for the formula

$$C1 \sqsubseteq \ (\geq 1 \ a_{C1}^{-})$$

expressing that a_{C1} is a role mandatory. Hence $\mathcal{J}$ satisfies the $\mathcal{ALCIN}$ formula, since by the FOL formula, every element $e \in C1^{\mathcal{I}}$ is in the relationship A, ie $(e,y) \in A^{\mathcal{I}}$, therefore there is at least one element $(t,e) \in a_{C1}^{\mathcal{J}}$ for every $e \in C1^{\mathcal{J}}$.

Object Type Subsumptions In this case, the following DL axiom is in $\Sigma^{zero}_{\mathcal{ALCIN}}$

$$CChild \sqsubseteq CParent$$

This encoding is derived from the embedding rule (**UML-1S**).

Thus a subsumption between the classes $Child$ and $Parent$ is in D^{zero}_{UML}. $\mathcal{I}$ satisfies the FOL formula

$$\forall x.CChild(x) \supset CParent(x)$$

and considering that for all UML classes, we have defined $C^{\mathcal{I}} = C^{\mathcal{J}}$, therefore, $CChild^{\mathcal{J}} \subseteq CParent^{\mathcal{J}}$.

Relationship subsumption The FOL formula is

$$\forall x \forall y.RelChild(x,y) \supset RelParent(x,y)$$

which is satisfied by $\mathcal{I}$.

This fórmula is encoded in KF metamodel through the rule (**UML-SA1**). By the way we have built the interpretation $\mathcal{J}$ for relationships, the DL encoding

$$RelChild \sqsubseteq RelParent$$

is satisfied.

Completeness constraint Subsumption- Object types Completeness Subsumption The following DL axiom is in $\Sigma^{zero}_{\mathcal{ALCIN}}$

$$C_1 \sqsubseteq CSup$$
$$C_2 \sqsubseteq CSup$$
$$\vdots$$
$$C_n \sqsubseteq CSup$$
$$CSup \sqsubseteq C_1 \sqcup C_2 \sqcup ... \sqcup C_n$$

This encoding is derived from the embedding rule (**UML-C1**), which can be extended to more than two classes.

As the FOL subsumption formulas are satisfied by $\mathcal{I}$, then the $\mathcal{ALCIN}$ subclass formulas are satisfied by $\mathcal{J}$.

Finally, the following FOL formula

$$\forall x.CSup(x) \supset \bigvee_{i=1}^{n} C_i(x), \text{ for } i = 1, \ldots, n-1$$

is satisfiable by $\mathcal{I}$, then every element in class $CSup$ must be in C_j for some $1 \leq j \leq n$. Therefore, the interpretation $\mathcal{J}$ satisfies:

$$CSup^{\mathcal{J}} \subseteq \bigcup_{i=1}^{n} C_i^{\mathcal{J}}$$

Object types Disjoint Subsumption The following DL axioms are in $\Sigma^{zero}_{\mathcal{ALCIN}}$

$$C_1 \sqsubseteq CSup$$
$$C_2 \sqsubseteq CSup$$
$$\vdots$$
$$C_n \sqsubseteq CSup$$
$$C_i \sqsubseteq \bigsqcap_{j=i+1}^{n} \neg C_j, \text{ for } i = 1, \ldots, n-1$$

This encoding has been derived from the embedding rule (**UML-D1**) The following FOL formulas capture the object types disjoint subsumption semantic

$$\forall x.C_1(x) \supset CSup(x)$$
$$\forall x.C_2(x) \supset CSup(x)$$
$$\vdots$$
$$\forall x.C_n(x) \supset CSup(x)$$
$$\forall x.C_i(x) \supset \bigwedge_{j=i+1}^{n} \neg C_j(x), \text{ for } i = 1, \ldots, n-1$$

By the definition of the interpretation $\mathcal{J}$ for concepts associated to classes, the $\mathcal{ALCIN}$ axioms are satisfied.

Hence, $\mathcal{J}$ is a model for $\Sigma^{zero}_{\mathcal{ALCIN}}$

($\Leftarrow$) By the tree-model property we know that if C is satisfiable w.r.t. the $\mathcal{ALCIN}$ knowledge base $\Sigma^{zero}_{\mathcal{ALCIN}}$ then there exists a tree-like model $\mathcal{J} = (\Delta^{\mathcal{J}}, .^{\mathcal{J}})$ of $\Sigma^{zero}_{\mathcal{ALCIN}}$, such that $C^{\mathcal{J}} \neq \varnothing$. From such a tree-like model we can build an instantiation $\mathcal{I} = (\Delta^{\mathcal{I}}, .^{\mathcal{I}})$ of D^{zero}_{UML} such that $C^{\mathcal{I}} \neq \varnothing$, as follows:

$\Delta^{\mathcal{I}} = \bigcup_{\mathbf{C} \in \mathcal{C}} \mathbf{C}^{\mathcal{J}}$, where $\mathcal{C}$ denotes the set of all classes in D^{zero}_{UML}.
$C^{\mathcal{I}} = C^{\mathcal{J}}$ for all classes C in D^{zero}_{UML}
Finally, for each binary association without association class, A, between $C1$ and $C2$ we define

$$A^{\mathcal{I}} = \{(d_{C1}, d_{C2}) | \exists t \in A^{\mathcal{J}}. \bigwedge_{Ci \in \mathcal{C_A}} (t, d_{Ci}) \in r^{\mathcal{J}}_{Ci}\}$$

where $\mathcal{C_A}$ is the set of the two concepts taking part in the relation A, ie$\{C1, C2\}$.

Since $\mathcal{J}$ is a tree-like model, it is guaranteed that there is only one object t in an objectified relation $A^{\mathcal{J}}$ representing a given tuple in A. Keeping such an observation in mind we must check that $\mathcal{I}$ is indeed an instantiation of D^{zero}_{UML} with $C^{\mathcal{I}} \neq \varnothing$.

Binary relationships A model $\mathcal{J}$ satisfies the following ALCIN assertion,

$$\exists a_{C1}.\top \sqsubseteq A$$
$$\exists a^{-}_{C1}.\top \sqsubseteq C1$$
$$\exists a_{C2}.\top \sqsubseteq A$$
$$\exists a^{-}_{C2}.\top \sqsubseteq C2$$

$$A \sqsubseteq \; \exists a_{C1} \sqcap (\leq 1 \; a_{C1}) \sqcap \exists a_{C2} \sqcap (\leq 1 \; a_{C2})$$

$$C1 \sqsubseteq \; (\geq card1 - min \; a^{-}_{C1}) \sqcap (\leq card1 - max \; a^{-}_{C1})$$
$$C2 \sqsubseteq \; (\geq card2 - min \; a^{-}_{C2}) \sqcap (\leq card2 - max \; a^{-}_{C2})$$

This encoding has been built from the KF metamodel applying the rules **UML-A1**, **UML-R1**,**UML-MC1**. Back to the UML diagram, D^{zero}_{UML} we recover a binary association A between two clases C1 and C2, with cardinality constraint card1-min and card1-max and card2-min and card2-max, respectively.

Thus, we must prove that $\mathcal{I}$ is a model of the following formulas:

$$\forall x, y.A(x, y) \supset C1(x) \wedge C2(y) \tag{1}$$
$$\forall x.C1(x) \supset (card1 - min \leq \#\{y|A(x, y)\} \leq card1 - max) \tag{2}$$
$$\forall y.C2(y) \supset (card2 - min \leq \#\{x|A(x, y)\} \leq card2 - max) \tag{3}$$

Each object $d \in \Delta^{\mathcal{J}}$ that is related via a role a_{C1} to an object ot_j, corresponding to a tuple in $A^{\mathcal{J}}$, is actually related to m objects $ot_j, 1 \leq j \leq m$, $card1 - min \leq m \leq card1 - max$, i.e. $(ot_j, d) \in a^{\mathcal{J}}_{C1}$, $1 \leq j \leq m$. Similarly, for the role a_{C2}.

By definition,

$$A^{\mathcal{I}} = \{(d_{C1}, d_{C2})|ot \in A^{\mathcal{J}} \wedge t = (d_{C1}, d_{C2}) \text{ is the correspondig tuple for the object } ot\}$$
$$C1^{\mathcal{I}} = C1^{\mathcal{J}}$$
$$C1^{\mathcal{I}} = C1^{\mathcal{J}}$$

The interpretation $\mathcal{I}$ built from $\mathcal{J}$ as above, populates the relation $A^{\mathcal{I}}$ with m tuples $t_1; \ldots; t_m$ corresponding to the objects in A, and such that ot_i corresponds to t_i for each $1 \leq i \leq m$. Thus, the elements in $A^{\mathcal{I}}$ are ordered pairs with first element in $C1$ and second en $C2$. Therefore, the equation (1) is satisfied.

Furthermore, according to the fact that $\mathcal{J}$ is a tree-like model, it is always possible to exclude the case where there is more than one tuple in $A^{\mathcal{I}}$ for each object in $A^{\mathcal{J}}$. Consequently, the formulas (2) and (3) are satisfied.

Mandatory role This case is considered as a binary relationship A in which one (or both) of the roles is/are mandatory. The DL encoding is the same as a binary relationship, except for the formula

$$C1 \sqsubseteq \; (\geq 1 \; a^{-}_{C1})$$

expressing that a_{C1} is a role mandatory. Hence $\mathcal{J}$ satisfies the $\mathcal{ALCIN}$ formula.

Object Type Subsumptions In this case, the following DL axiom is in $\Sigma^{zero}_{\mathcal{ALCIN}}$

$$CSub \sqsubseteq CSup$$

As $\mathcal{J}$ is a model of $\Sigma^{zero}_{\mathcal{ALCIN}}$, then $CSub^{\mathcal{J}} \subseteq CSup^{\mathcal{J}}$ is satisfied.
As we just consider the case that this subsumption between the classes $CSub$ and $CSup$ are in D^{zero}_{UML} and that this encoding is derived from the embedding rule (**UML-1S**)
Thus, the FOL formula

$$\forall x.CSub(x) \supset CSup(x)$$

is satisfied since we have defined $C^{\mathcal{I}} = C^{\mathcal{J}}$, for all UML classes, . Therefore, $CSub^{\mathcal{I}} \subseteq CSup^{\mathcal{I}}$.

Relationship subsumption $\mathcal{ALCIN}$ assertion

$$RelChild \sqsubseteq RelParent$$

is satisfied by $\mathcal{J}$.
This fórmula was encoded in KF metamodel through the rule (**UML-SA1**). Thus the corresponding FOL formula is

$$\forall x \forall y.RelChild(x, y) \supset RelParent(x, y)$$

which is satisfied by $\mathcal{I}$.

Completeness constraint Subsumption- Object types Completeness Subsumption The following DL axiom is in $\Sigma^{zero}_{\mathcal{ALCIN}}$ are satisfied by $\mathcal{J}$:

$$\begin{aligned} C_1 &\sqsubseteq CSup \\ C_2 &\sqsubseteq CSup \\ &\vdots \\ C_n &\sqsubseteq CSup \\ CSup &\sqsubseteq C_1 \sqcup C_2 \sqcup ... \sqcup C_n \end{aligned}$$

Therefore, it is satisfied the following:

$$\begin{aligned} C_1^{\mathcal{J}} &\subseteq CSup^{\mathcal{J}} \\ C_2^{\mathcal{J}} &\subseteq CSup^{\mathcal{J}} \\ &\vdots \\ C_n^{\mathcal{J}} &\subseteq CSup^{\mathcal{J}} \\ CSup^{\mathcal{J}} &\subseteq C_1^{\mathcal{J}} \cup C_2^{\mathcal{J}} \cup ... \cup C_n^{\mathcal{J}} \qquad (4) \end{aligned}$$

This encoding is derived from the following embedding rule (**UML-C1**). Thus, all the classes $CSup, C_1, \ldots, C_n$ are in D^{zero}_{UML} and for definition of $\mathcal{I}$ the FOL subsumption formulas are satisfied.
The following FOL formula

$$\forall x.CSup(x) \supset \bigvee_{i=1}^{n} C_i(x), \text{ for } i = 1, \ldots, n-1$$

is satisfied by $\mathcal{I}$, because the relation (4) is satisfied.

Object types Disjoint Subsumption The following DL axioms are in $\Sigma^{zero}_{\mathcal{ALCIN}}$ and they are all satisfied by $\mathcal{J}$

$$C_1 \sqsubseteq CSup$$
$$C_2 \sqsubseteq CSup$$
$$\vdots$$
$$C_n \sqsubseteq CSup$$
$$C_i \sqsubseteq \bigsqcap_{j=i+1}^{n} \neg C_j, \text{ for } i = 1, \ldots, n-1$$

We consider the case where this encoding has been built from the KF embedding rule (**UML-D1**). So the following FOL formulas must been satisfied by $\mathcal{I}$:

$$\forall x.C_1(x) \supset CParent(x)$$
$$\forall x.C_2(x) \supset CParent(x)$$
$$\vdots$$
$$\forall x.C_n(x) \supset CParent(x)$$
$$\forall x.C_i(x) \supset \bigwedge_{j=i+1}^{n} \neg C_j(x), \text{ for } i = 1, \ldots, n-1$$

As every class in this formulas are in D^{zero}_{UML} then above formulas are satisfied by $\mathcal{I}$.

Hence, $\mathcal{I}$ is a model for D^{zero}_{UML}.

□

3 ORM 2

Definition 3. *Let $ORM2^{zero}$ be the fragment of ORM2 that includes the following primitives:*

- *Entity type*
- *Role*
- *Binary Fact Type*
- *Frequency constraints Frecuency*
- *Subset constraint on fact type*
- *Total*
- *Exclusive*

We define D^{zero}_{ORM2} as $ORM2^{zero}$ conceptual model.

Theorem 2. *Let D^{zero}_{ORM2} be an ORM^{zero} diagram, Σ^{zero} be the corresponding KF conceptual schema built using the embedding rules and $\Sigma^{zero}_{\mathcal{ALCIN}}$ the $\mathcal{ALCIN}$ knowledge base constructed as described in Table 2.*
An entity type E is consistent in D^{zero}_{ORM} if and only if the corresponding concept encoding of E, is satisfiable in $\Sigma^{zero}_{\mathcal{ALCIN}}$.

Proof. We assume that the signatures of the symbols representing entity types, fact types and roles are disjoint.

In the scope of this proof we will consider the FOL formalisation of ORM2 in [3] as semantics of D^{zero}_{ORM2}.

($\Rrightarrow$) Given a model $\mathcal{I} = (\Delta^{\mathcal{I}}, .^{\mathcal{I}})$ be an instantiation of D^{zero}_{ORM2}, ie. a model of the corresponding FOL assertions, such that $E^{\mathcal{I}} \neq \varnothing$. Then we can build a model $\mathcal{J} = (\Delta^{\mathcal{J}}, .^{\mathcal{J}})$ of $\Sigma^{zero}_{\mathcal{ALCIN}}$ such that $E^{\mathcal{J}} \neq \varnothing$ as follows:

$\Delta^{\mathcal{J}} = \Delta^{\mathcal{I}} \cup \bigcup_{FT \in \mathcal{FT}} \{t_{(d_{E1},d_{E2})} | (d_{E1}, d_{E2}) \in FT^{\mathcal{I}}\}$ where $\mathcal{FT}$ denotes the set of all binary fact type.

$E^{\mathcal{J}} = E^{\mathcal{I}}$ for each concept E correponding to entity type in D^{zero}_{UML}

Finally, for each binary fact type FT, we define $FT^{\mathcal{J}} = \{t_{(d_{E1},d_{E2})} | (d_{E1}, d_{E2}) \in FT^{\mathcal{I}}\}$ and for each $\mathcal{ALCIN}$ role modeling de ith-component of the fact type FT, we define $ft_i^{\mathcal{J}} = \{(t_{(d_{E1},d_{E2})}, d_{Ei}) | (d_{E1}, d_{E2}) \in FT^{\mathcal{I}}\}$

Trivially, $E^{\mathcal{J}} = E^{\mathcal{I}} \neq \varnothing$. As for the rest of the expressions in Σ^{zero}, it must be verified that for all $\mathcal{I}$ that are model of Σ^{zero}_{ORM2}, there is a $\mathcal{J}$ that is a model of the corresponding $\mathcal{ALCIN}$ knowledge base.

Binary Fact Type $\mathcal{I}$ is a model for Fact Type(P(E1,E2)).Thus the formula

$$\forall xy.P(x, y) \supset E1(x) \wedge E2(y)$$

is satisfied.

A binary fact type is encoded in KF metamodel as a binary relationship reified with two roles(**ORM2-O1**,**ORM2-A1**, **ORM2-R1**). This encoding is then translate to DL as follow:

$$\exists p_{E1} \sqsubseteq P$$
$$\exists p^-_{E1} \sqsubseteq E1$$
$$\exists p_{E2} \sqsubseteq P$$
$$\exists p^-_{E2} \sqsubseteq E2$$

$$P \sqsubseteq \exists p_{E1} \sqcap (\leq 1\ p_{E1}) \sqcap \exists p_{E2} \sqcap (\leq 1\ p_{E2})$$

By definition,

$$P^{\mathcal{J}} = \{t_{(d_{E1},d_{E2})} | (d_{E1}, d_{E2}) \in P^{\mathcal{I}}\}$$
$$p^{\mathcal{J}}_{E1} = \{(t_{(d_{E1},d_{E2})}, d_{E1}) | (d_{E1}, d_{E2}) \in P^{\mathcal{I}}\}$$
$$p^{\mathcal{J}}_{E2} = \{(t_{(d_{E1},d_{E2})}, d_{E2}) | (d_{E1}, d_{E2}) \in P^{\mathcal{I}}\}$$

Therefore, $\mathcal{J}$ is a model for the first four $\mathcal{ALCIN}$ formulas, because they express the domain and range of roles p_{E1} and p_{E2}.

The fifth DL formula express that there is just one pair (d_{E1}, d_{E2}) in the P concept and that this pair is in both roles, which is true because of the definition of the interpretation $\mathcal{J}$ for roles and fact type.

Frequency constraints We consider four types of Frecuency(P_i, $\underline{F}$), where Fact Type(P(E1,E2)) and

(1) $\underline{F}$ is (min..max)

(i) $min > 1$: If i=1 then

$$\forall x_1, x_2.P(x_1, x_2) \supset \exists^{\geq min} y P(x_1, y) \wedge \exists^{\leq max} y P(x_1, y) \tag{5}$$

Similarly, when i=2.

In this case, the role p_{E1} must be mandatory, in order to be in the conditions of the KF metamodel rule ORM2-MC1-2. Therefore, $\mathcal{I}$ satisfies the formula above and

$$\forall x.E1(x) \supset \exists y P(x, y) \tag{6}$$

The following $\mathcal{ALCIN}$ formula is generated from ORM2-MC1

$$E1 \sqsubseteq (\geq min\ p^{-}_{E1}) \sqcap (\leq max\ p^{-}_{E1}) \tag{7}$$

By the formula (6), forall $e1 \in E1^{\mathcal{I}}$, exists at least one $e' \in E2^{\mathcal{I}}$, such that $(e1, e') \in P^{\mathcal{I}}$. By equation (5) there exists $y_1, \ldots, y_m$ such that $(o, y_m) \in P^{\mathcal{I}}$ and $min \leq m \leq max$.
By definition

$$p^{\mathcal{J}}_{E1} = \{(t(d_{E1}, d_{E2}), d_{E1}) | (d_{E1}, d_{E2}) \in P^{\mathcal{I}}\}$$

Thus, the following formulas $(\geq min\ p^{-}_{E1})$ and $(\leq max\ p^{-}_{E1})$ are satisfied $\forall e1 \in E1^{\mathcal{J}}$.
$\therefore$ The assertion (7) is satisfied by $\mathcal{J}$.

(ii) $min = 1$: If i=1 then

$$\forall x_1, x_2.P(x_1, x_2) \supset \exists^{\geq 1} y P(x_1, y) \wedge \exists^{\leq max} y P(x_1, y) \tag{8}$$

and $\mathcal{I}$ satisfies the formula above. Similarly, when i=2.
The KF rule applied is ORM2-MC1-1. The encoded range is $(0, max)$ and the following $\mathcal{ALCIN}$ formula is generated

$$E1 \sqsubseteq (\leq max\ p^{-}_{E1}) \tag{9}$$

By equation (8) if two elements are related by P, then they must satisfy the frequency constraint under $\mathcal{I}$.
By definition

$$p^{\mathcal{J}}_{E1} = \{(t(d_{E1}, d_{E2}), d_{E1}) | (d_{E1}, d_{E2}) \in P^{\mathcal{I}}\}$$

Thus, the formula $(\leq max\ p^{-}_{E1})$ is satisfied $\forall e1 \in E1^{\mathcal{J}}$, since if $e1$ is not in the relationship P, then the frecuency is 0 and if $e1$ is in the relationship then it must satisfy the cardinality.
$\therefore$ The assertion (9) is satisfied by $\mathcal{J}$.

(iii) $min = 0$: If i=1 then

$$\forall x_1, x_2.P(x_1, x_2) \supset \exists^{\leq max} y P(x_1, y) \tag{10}$$

and $\mathcal{I}$ satisfies the formula above. Similarly, when i=2.
The KF rule applied is ORM2-MC1-1. The encoded range is $(0, max)$ and the following $\mathcal{ALCIN}$ formula is generated

$$E1 \sqsubseteq \ (\leq max \ p^{-}_{E1}) \tag{11}$$

By equation (10), every element in $E1$ that is related by P, must satisfy the frequency constraint under $\mathcal{I}$.
By definition

$$p^{\mathcal{J}}_{E1} = \{(t(d_{E1}, d_{E2}), d_{E1}) | (d_{E1}, d_{E2}) \in P^{\mathcal{I}}\}$$

Thus, the formula $(\leq max \ p^{-}_{E1})$ is satisfied $\forall e1 \in E1^{\mathcal{J}}$, since if $e1$ is not in the relationship P, then the frecuency is 0 and if $e1$ is in the relationship then it must satisfy the cardinality.
$\therefore$ The assertion (11) is satisfied by $\mathcal{J}$.

(2) $\underline{F}$ is (min..): If i=1 then

$$\forall x_1, x_2.P(x_1, x_2) \supset \exists^{\geq min} y P(y, x_2) \tag{12}$$

Similarly, when i=2.

(i) If $min = 1$ then

$$\forall x_1, x_2.P(x_1, x_2) \supset \exists^{\geq 1} y P(y, x_2)$$

This case is similarly to (1)(ii)

(ii) If $min > 1$, the role p_{E1} must be mandatory, in order to be in the conditions of the KF metamodel rule ORM2-MC1-2. Therefore, $\mathcal{I}$ satisfies the formula (12) and

$$\forall x.E1(x) \supset \exists y P(x, y) \tag{13}$$

The following $\mathcal{ALCIN}$ formula is generated from ORM2-MC1

$$E1 \sqsubseteq \ (\geq min \ p^{-}_{E1}) \tag{14}$$

By the formula (13), forall $e1 \in E1^{\mathcal{I}}$, exists at least one $e' \in E2^{\mathcal{I}}$, such that $(e1, e') \in P^{\mathcal{I}}$. By the formula (12) there exists $y_1, \ldots, y_m$ such that $(o, y_m) \in P^{\mathcal{I}}$ and $min \leq m$.
By definition

$$p^{\mathcal{J}}_{E1} = \{(t(d_{E1}, d_{E2}), d_{E1}) | (d_{E1}, d_{E2}) \in P^{\mathcal{I}}\}$$

Thus, the following formula $(\geq min \ p^{-}_{E1})$ is satisfied $\forall e1 \in E1^{\mathcal{J}}$.
$\therefore$ The assertion (14) is satisfied by $\mathcal{J}$.

(3) $\underline{F}$ is (..max): If i=1 then

$$\forall x_1, x_2.P(x_1, x_2) \supset \exists^{\leq max} y P(y, x_2)$$

Similarly, when i=2.
This case is similarly to (1)(iii)

(4) $\underline{F}$ is (card): If i=1 then

$$\forall x_1, x_2.P(x_1, x_2) \supset \exists^{=card} y P(y, x_2)$$

Similarly, when i=2.
This case can be considered as $\underline{F}$ (card, card)

Subtype $\mathcal{I}$ satisfies

$$\forall x.E1(x) \supset E(x)$$

In this case, the following DL axiom is in $\Sigma^{zero}_{\mathcal{ALCIN}}$

$$E1 \sqsubseteq E$$

for $E, E1$ entity types. This encoding is derived from the following embedding rule ORM2-S1.

By the way we built the interpretation$\mathcal{J}$ for entity types, it satisfies the DL axiom.

Subset constraint on fact type Applies only to a pair offact types.
$\mathcal{I}$ satisfies

$$\forall x, y.P(x, y) \supset P'(x, y)$$

In this case, the following DL axiom is in $\Sigma^{zero}_{\mathcal{ALCIN}}$

$$P \sqsubseteq P'$$

This encoding is derived from the following embedding rule ORM2-SA1.

By the definition of the interpretation $\mathcal{J}$ for fact types, we can conclude that it satisfies the DL axiom.

Simple Mandatory $\mathcal{I}$ satisfies Mand(E1,P$_i$):If $i = 1$, $\forall x.E1(x) \supset \exists y P(x, y)$. Similarly, when $i = 2$. Thus the relationship has at least one tuple for every element in $E1(x)$.

Consider P the relationship and p_{E1} the role between P and E1, which is mandatory. Then the embedding rule ORM2-M1 has been applied and the following DL generated.

$$E1 \sqsubseteq \geq 1\ p^{-}_{E1}$$

By definition, $p^{\mathcal{J}}_{E1}$ there is at least one element (t, e) for every $e \in E1$. Therefore, the DL axiom is satisfied by $\mathcal{J}$.

Total $\mathcal{I}$ satisfies Exhaustive Subtype({E1,E2,. . . , En}; E):

$$\Big((\forall x.E1(x) \supset E(x)) \wedge (\forall x.E2(x) \supset E(x)) \ldots (\forall x.En(x) \supset E(x))\Big) \wedge (\forall x.E(x) \supset E1(x) \vee \ldots \vee En(x))$$

The following DL axiom is in $\Sigma^{zero}_{\mathcal{ALCIN}}$

$$E_1 \sqsubseteq E$$
$$E_2 \sqsubseteq E$$
$$\vdots$$
$$E_n \sqsubseteq E$$
$$E \sqsubseteq E_1 \sqcup E_2 \sqcup ... \sqcup E_n$$

This encoding is derived from the embedding rule(**ORM2-C1**).

By definition of $\mathcal{J}$ for the entity types in D^{zero}_{ORM2}, it satisfies the $\mathcal{ALCIN}$ formulas.

Exclusive $\mathcal{I}$ satisfies ExclusiveSubtype({E1,E2,..., En}; E):

$$\Big(\forall x.E1(x) \supset E(x) \wedge \neg E2(x) \wedge \ldots \wedge \neg En(x)\Big) \wedge \ldots \wedge \Big(\forall x.En{-}1(x) \supset E(x) \wedge \neg En(x)\Big) \wedge \Big(\forall x.En(x) \supset E(x$$

The following DL axioms are in $\Sigma^{zero}_{\mathcal{ALCIN}}$

$$E_1 \sqsubseteq E$$
$$E_2 \sqsubseteq E$$
$$\vdots$$
$$E_n \sqsubseteq E$$
$$E_i \sqsubseteq \prod_{j=i+1}^{n} \neg E_j, \text{ for } i = 1, \ldots, n-1$$

This encoding is derived from the embedding rule(**ORM2-D1**).

By definition of $\mathcal{J}$ for the entity types in D^{zero}_{ORM2}, it satisfies the $\mathcal{ALCIN}$ formulas.

Hence, $\mathcal{J}$ is a model of $\Sigma^{zero}_{\mathcal{ALCIN}}$.

($\Leftarrow$) By the tree-model property we know that if E is satisfiable w.r.t. the $\mathcal{ALCIN}$ knowledge base $\Sigma^{zero}_{\mathcal{ALCIN}}$ then there exists a tree-like model $\mathcal{J} = (\Delta^{\mathcal{J}}, .^{\mathcal{J}})$ of $\Sigma^{zero}_{\mathcal{ALCIN}}$, such that $E^{\mathcal{J}} \neq \varnothing$. From such a tree-like model we can build an instantiation $\mathcal{I} = (\Delta^{\mathcal{I}}, .^{\mathcal{I}})$ of D^{zero}_{ORM2} such that $E^{\mathcal{I}} \neq \varnothing$, as follows:

$\Delta^{\mathcal{I}} = \bigcup_{\mathbf{E} \in \mathcal{E}} \mathbf{E}^{\mathcal{J}}$, where $\mathcal{E}$ denotes the set of all entities in D^{zero}_{ORM2}.

$C^{\mathcal{I}} = C^{\mathcal{J}}$ for all entity types E in D^{zero}_{ORM2}

Finally, for each binary relationship, P, we define

$$P^{\mathcal{I}} = \{(d_{E1}, d_{E2}) | \exists t \in P^{\mathcal{J}}. \bigwedge_{j=1}^{2} (t, d_E j) \in r_E j^{\mathcal{J}}\}$$

Since $\mathcal{J}$ is a tree-like model, it is guaranteed that there is only one object t in an objectified relation $P^{\mathcal{J}}$ representing a given tuple in P. Keeping such an observation in mind we must check that $\mathcal{I}$ is indeed an instantiation of D^{zero}_{ORM2} with $E^{\mathcal{I}} \neq \varnothing$.

Binary Fact Type A binary fact type is encoded in KF metamodel as a binary relationship reified with two roles(**ORM2-O1**,**ORM2-A1**, **ORM2-R1**). This encoding is then translate to DL as follow:

$$\exists p_{E1}.\top \sqsubseteq P$$
$$\exists p^{-}_{E1}.\top \sqsubseteq E1$$
$$\exists p_{E2}.\top \sqsubseteq P$$
$$\exists p^{-}_{E2}.\top \sqsubseteq E2$$

$$P \sqsubseteq \exists p_{E1} \sqcap (\leq 1\ p_{E1}) \sqcap \exists p_{E2} \sqcap (\leq 1\ p_{E2})$$

$\mathcal{J}$ is a model for the $\mathcal{ALCIN}$ formulas
The corresponding FOL formula is

$$\forall xy.P(x,y) \supset E1(x) \wedge E2(y)$$

By definition,

$$P^{\mathcal{I}} = \{(d_{E1}, d_{E2}) | ot \in P^{\mathcal{J}} \wedge t = (d_{E1}, d_{E2}) \text{ is the correspondig tuple for the object } ot\}$$
$$E1^{\mathcal{I}} = E1^{\mathcal{J}}$$
$$E2^{\mathcal{I}} = E2^{\mathcal{J}}$$

The interpretation $\mathcal{I}$ built from $\mathcal{J}$ as above, populates the relation $P^{\mathcal{I}}$ with m tuples $t_1; \ldots; t_m$ corresponding to the objects in P, and such that ot_i corresponds to t_i for each $1 \leq i \leq m$. Thus, the elements in $P^{\mathcal{I}}$ are ordered pairs with first element in $E1$ and second en $E2$. Therefore, the FOL formula is satisfied by $\mathcal{I}$.

Frequency constraints We consider four types of Frecuency(P_i, $\underline{F}$), where Fact Type(P(E1,E2)) and

(1) $\underline{F}$ is (min..max): If i=1 then

$$\forall x_1, x_2.P(x_1, x_2) \supset \exists^{\geq min} y P(x_1, y) \wedge \exists^{\leq max} y P(x_1, y) \tag{15}$$

Similarly, when i=2.

(i) $min > 1$: The following $\mathcal{ALCIN}$ formula is generated from ORM2-MC2 and is satisfied by $\mathcal{I}$.

$$E1 \sqsubseteq (\geq min\ p^{-}_{E1}) \sqcap (\leq max\ p^{-}_{E1}) \tag{16}$$

In this case, the KF metamodel rule ORM2-MC1-2 has been applied. Thus $\mathcal{J}$ must satisfy the formula (15) and the mandatory expression

$$\forall x.E1(x) \supset \exists y P(x,y) \tag{17}$$

By definition

$$P^{\mathcal{I}} = \{(d_{E1}, d_{E2}) | \exists t \in P^{\mathcal{J}}. \bigwedge_{j=1}^{2} (t, d_{Ej}) \in p^{\mathcal{J}}_{Ej}\}$$

Knowing that $\mathcal{I}$ satisfies (16) and considering that $min > 1$ and the definition of $\mathcal{J}$ we can conclude that every element $e \in E1$ is in relation P with at least one element. Thus the formula (17), is satisfied by $\mathcal{J}$.
Furthermore, by the definition of $P^{\mathcal{I}}$ and (16) for every $e \in E1$, there exists $y_1, \ldots, y_m$ such that $(e, y_m) \in P^{\mathcal{I}}$ and $min \leq m \leq max$.

$\therefore$ The assertion (15) is satisfied by $\mathcal{I}$.

(ii) If i=1 then the KF rule applied is ORM2-MC1-1. The encoded range is $(0, max)$ and the following $\mathcal{ALCIN}$ formula is satisfied by $\mathcal{J}$

$$E1 \sqsubseteq (\leq max\ p^{-}_{E1}) \tag{18}$$

Thus, every element $e \in E1^{\mathcal{J}}$, if there is no tuple $(e, y) \in P^{\mathcal{J}}$ then (15) is satisfied by $\mathcal{I}$.
If e participes in the relationship P, then it must satisfy cardinality constraints. Therefore, by the definition of $P^{\mathcal{I}}$, (15) is satisfied by $\mathcal{I}$.

(iii) $min = 0$: This case is similar to the above case.

(2) $\underline{F}$ is (min..): If i=1 then

$$\forall x_1, x_2.P(x_1, x_2) \supset \exists^{\geq v} y P(y, x_2)$$

Similarly, when i=2.
This case is similarly to (1)(ii) when $min = 1$ and to (1)(i) when $min > 1$

(3) $\underline{F}$ is (..max): If i=1 then

$$\forall x_1, x_2.P(x_1, x_2) \supset \exists^{\leq w} y P(y, x_2)$$

Similarly, when i=2.
This case is similarly to (1)(iii)

(4) $\underline{F}$ is (card): If i=1 then

$$\forall x_1, x_2.P(x_1, x_2) \supset \exists^{= v} y P(y, x_2)$$

Similarly, when i=2.
This case can be considered as $\underline{F}$ (card..card).

Subtype $\mathcal{J}$ satisfies

$$E1 \sqsubseteq E$$

the DL axiom is in $\Sigma^{zero}_{\mathcal{ALCIN}}$
This encoding is derived from the embedding rule ORM2-S1.
By the way we built the interpretation $\mathcal{I}$ for entity types, it satisfies the FOL formula

$$\forall x.E1(x) \supset E(x)$$

Subset constraint on fact type Applies only to a pair of fact types.
The $\mathcal{ALCIN}$ axiom in $\Sigma^{zero}_{\mathcal{ALCIN}}$ is

$$P \sqsubseteq P'$$

and it is satisfies by $\mathcal{J}$.
This encoding is derived from the following embedding rule ORM2-SA1.
By the definition of the interpretation $\mathcal{I}$ for fact types, we can conclude that it satisfies

$$\forall x, y.P(x, y) \supset P'(x, y)$$

Simple Mandatory Consider P the relationship and p_{E1} the role between P and E1, which is mandatory. Then the embedding rule ORM2-M1 has been applied and the following DL generated.

$$E1 \sqsubseteq \geq 1 \; p^{-}_{E1}$$

As $\mathcal{J}$ satisfies the above formula, so $p^{\mathcal{J}}_{E1}$ has at least one element (t, e) for every $e \in E1$. Thus the relationship $P^{\mathcal{I}}$ has at least one tuple for every element in $E1(x)$.
Therefore, the FOL formula Mand(E,P$_i$):If $i = 1$, $\forall x.E(x) \supset \exists y P(x, y)$ is satisfied by $\mathcal{I}$.

Total The following DL axioms are in $\Sigma^{zero}_{\mathcal{ALCIN}}$

$$E_1 \sqsubseteq E$$
$$E_2 \sqsubseteq E$$
$$\vdots$$
$$E_n \sqsubseteq E$$
$$E \sqsubseteq E_1 \sqcup E_2 \sqcup ... \sqcup E_n$$

and are satisfied by $\mathcal{J}$. This encoding is derived from the embedding rule **ORM2-C1**. By definition of $\mathcal{I}$ for the entity types in D^{zero}_{ORM2}, it satisfies the following FOL formula: Exhaustive Subtype({E1,E2,. . . , En}; E)

$$\Big((\forall x.E1(x) \supset E(x)) \wedge (\forall x.E2(x) \supset E(x)) \ldots (\forall x.En(x) \supset E(x))\Big) \wedge (\forall x.E(x) \supset E1(x) \vee \ldots \vee En(x))$$

Exclusive $\mathcal{J}$ satisfies the following DL axioms in $\Sigma^{zero}_{\mathcal{ALCIN}}$

$$E_1 \sqsubseteq E$$
$$E_2 \sqsubseteq E$$
$$\vdots$$
$$E_n \sqsubseteq E$$
$$E_i \sqsubseteq \prod_{j=i+1}^{n} \neg E_j, \text{ for } i = 1, \ldots, n-1$$

This encoding is derived from the embedding rule **ORM2-D1**. Thus, by definition of $\mathcal{I}$ for the entity types in D^{zero}_{ORM2}, the following FOL formula is satisfied: ExclusiveSubtype({E1,E2,. . . , En}; E):

$$\Big(\forall x.E1(x) \supset E(x) \wedge \neg E2(x) \wedge \ldots \wedge \neg En(x)\Big) \wedge \ldots \wedge \Big(\forall x.En{-}1(x) \supset E(x) \wedge \neg En(x)\Big) \wedge \Big(\forall x.En(x) \supset E(x$$

Hence, $\mathcal{I}$ is a model of D^{zero}_{ORM2}.

□

4 EER

Definition 4. *Let EER^{zero} be the fragment of EER that includes the following primitives:*

- *Entities*
- *Binary relationships*
- *Component of a relation*
- *Cardinality constraint*
- *Subtype*

- *Subtyping of Relationship*
- *Mandatory relationship*
- *Entities completeness constraints Subsumption*
- *Entities disjoint subsumption*

We define D^{zero}_{EER} as EER^{zero} conceptual model.

Theorem 3. *Let D^{zero}_{EER} be an EER^{zero} diagram, Σ^{zero} be the corresponding KF conceptual schema built using the interoperabilities rules and $\Sigma^{zero}_{\mathcal{ALCIN}}$ the $\mathcal{ALCIN}$ knowledge base constructed as described in Table 2.*
An entity E is consistent in D^{zero}_{EER} if and only if the corresponding concept encoding of E, is satisfiable in $\Sigma^{zero}_{\mathcal{ALCIN}}$.

Proof. We assume that the signatures of the symbols representing entities, relationships and roles are disjoint.

In the scope of this proof we will consider the formalisation of EER in [4] as semantics of D^{zero}_{EER}.

(⇒) Let $\mathcal{I} = (\Delta^{\mathcal{I}}, .^{\mathcal{I}})$ be an instantiation of D^{zero}_{EER}, ie. a model of the corresponding semantics, such that $E^{\mathcal{I}} \neq \varnothing$. Then we can build a model $\mathcal{J} = (\Delta^{\mathcal{J}}, .^{\mathcal{J}})$ of $\Sigma^{zero}_{\mathcal{ALCIN}}$ such that $E^{\mathcal{J}} \neq \varnothing$ as follows:

$\Delta^{\mathcal{J}} = \Delta^{\mathcal{I}} \cup \bigcup_{R \in \mathcal{R}} \{t_{(e1,e2)} | (e1, e2) \in \mathcal{R}^{\mathcal{I}}\}$ where $\mathcal{R}$ denotes the set of all binary relationships.

$E^{\mathcal{J}} = E^{\mathcal{I}}$ for each concept E correponding to entities in D^{zero}_{EER}

For each binary relationship R, we define $R^{\mathcal{J}} = \{t_{(e1,e2)} | (e1, e2) \in R^{\mathcal{I}}\}$.

For each $\mathcal{ALCIN}$ role modeling the ith-component of the binary relationship R, we define

$$r^{\mathcal{J}}_{E1} = \{(t_{(e1,e2)}, e_1) | (e1, e2) \in R^{\mathcal{I}}\}$$

and

$$r^{\mathcal{J}}_{E2} = \{(t_{(e1,e2)}, e_2) | (e1, e2) \in R^{\mathcal{I}}\}$$

Trivially, $E^{\mathcal{J}} = E^{\mathcal{I}} \neq \varnothing$. As for the rest of the expressions in Σ^{zero}, it must be verified that for all $\mathcal{I}$ that are model of Σ^{zero}_{EER}, there is a $\mathcal{J}$ that is a model of the corresponding $\mathcal{ALCIN}$ knowledge base.

Binary relationships $\mathcal{I}$ satisfies the following condition: For each relationship $R = \langle r_{E1} : E_1, r_{E2} : E_2 \rangle$, then

$$\forall (e_1, e_2) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}. (e_1, e_2) \in R^{\mathcal{I}} \rightarrow e_1 \in E_1 \land e_2 \in E_2$$

In this case, the KF embedding rule applied is ER-A1.

This rule is encoded in $\mathcal{ALCIN}$ as

$$\exists r_{E1}.\top \sqsubseteq R$$

$$\exists r^{-}_{E1}.\top \sqsubseteq E1$$

$$\exists r_{E2}.\top \sqsubseteq R$$

$$\exists r^{-}_{E2}.\top \sqsubseteq E2$$

$$R \sqsubseteq \exists r_{E1} \sqcap (\leq 1\ r_{E1}) \sqcap \exists r_{E2} \sqcap (\leq 1\ r_{E2})$$

By definition,

$$R^{\mathcal{J}} = \{t_{(e1,e2)}|(e1,e2) \in R^{\mathcal{I}}\}$$
$$r_{E1}^{\mathcal{J}} = \{(t_{(e1,e2)}, e_1)|(e1,e2) \in R^{\mathcal{I}}\}$$
$$r_{E2}^{\mathcal{J}} = \{(t_{(e1,e2)}, e_2)|(e1,e2) \in R^{\mathcal{I}}\}$$

Therefore, $\mathcal{J}$ is a model for the first four $\mathcal{ALCIN}$ formulas, because they express the domain and range of roles $r_{E1},\ r_{E2}$.

Finally, the fifth DL formula express that there is just one pair $(e1, e2)$ in the R class and that this pair is in both roles, which is true because of the definition of $\mathcal{J}$.

Cardinality constraint $\mathcal{I}$ satisfies the following condition: For each relationship $R = \langle r_{E1} : E_1, r_{E2} : E_2\rangle$, then

$$CARD_R(R, r_{E1}, E_1) = (min1, max1) \rightarrow \forall e_1 \in E_1^{\mathcal{I}}.min1 \leq |(e_1, e) \in R^{\mathcal{I}}| \leq max1$$

and

$$CARD_R(R, r_{E2}, E_2) = (min2, max2) \rightarrow \forall e_2 \in E_2^{\mathcal{I}}.min2 \leq |(e, e_2) \in R^{\mathcal{I}}| \leq max2$$

The KF embedding rule applied is **ER-MC1**, which generates the following $\mathcal{ALCIN}$ formulas

$$E_1 \sqsubseteq\ (\geq min1\ r_{E1}^{-}) \sqcap (\leq max1\ r_{E1}^{-})$$

$$E_2 \sqsubseteq\ (\geq min2\ r_{E2}^{-}) \sqcap (\leq max2\ r_{E2}^{-})$$

By definition,

$$r_{E1}^{\mathcal{J}} = \{(t_{(e1,e2)}, e_1)|(e1,e2) \in R^{\mathcal{I}}\}$$
$$r_{E2}^{\mathcal{J}} = \{(t_{(e1,e2)}, e_2)|(e1,e2) \in R^{\mathcal{I}}\}$$

Thus the tuples in each role $r_{Ei}^{\mathcal{J}}$ satisfied the same cardinality constraints that $R^{\mathcal{I}}$ and therefore $\mathcal{J}$ satisifies the $\mathcal{ALCIN}$ formulas.

Subtype Let E_1 and E_2 two entities. $\mathcal{I}$ satisfies: E_1 ISA E_2 implies $E_1^{\mathcal{I}} \subseteq E_2^{\mathcal{I}}$

In this case, the following DL axiom in $\Sigma_{\mathcal{ALCIN}}^{zero}$ is

$$E_2 \sqsubseteq E_1$$

This encoding is derived from the KF embedding rule **ER-S1**. Thus a subsumption between the entities E_2 and E_1 is in D_{EER}^{zero}. As $\mathcal{I}$ is a model of E_1 ISA E_2 and for all entities, we have defined $E^{\mathcal{I}} = E^{\mathcal{J}}$. Therefore, $E_1^{\mathcal{J}} \sqsubseteq E_2^{\mathcal{J}}$.

Subtyping of Relationship Let R_1 and R_2 two binary relationships. $\mathcal{I}$ satisfies R_1 ISA R_2 implies $R_1^{\mathcal{I}} \subseteq R_2^{\mathcal{I}}$

This fórmula is encoded in KF metamodel through the KF interaoperability rule **ER-SA1**.

By definition of the interpretation $\mathcal{J}$ for relationship, it satisfies the DL encoding

$$R_1 \sqsubseteq R_2$$

.

Mandatory relationship For each relationship $R = \langle r_{E1} : E_1, r_{E2} : E_2 \rangle$, then

$$CARD_R(R, r_{E1}, E_1) = (1, max1) \to \forall e_1 \in E_1^{\mathcal{I}}.1 \leq |(e_1, e) \in R^{\mathcal{I}}| \leq max1$$

This assertion expresses that for every element in the entity E_1, it must be the case that is in relation R. Thus the DL encoding

$$E1 \sqsubseteq \ (\geq 1\ r_{E1}^{-})$$

expressing that r_{E1} is a role mandatory is satisfied by $\mathcal{J}$.

Entities completeness constraints Subsumption $\mathcal{I}$ satisfies the following condition: For each entity $E, E_1, \ldots, E_n$, $\{E_1, \ldots, E_n\}$COV E implies $E_i^{\mathcal{I}} \subseteq E^{\mathcal{I}}, 1 \leq i \leq n$ and $E^{\mathcal{I}} \subseteq \bigcup_{i=1}^{n} E_i^{\mathcal{I}}$
The embedding rule **ER-C1** is applied and the following DL axioms are in $\Sigma_{\mathcal{ALCIN}}^{zero}$

$$E_1 \sqsubseteq E$$
$$E_2 \sqsubseteq E$$
$$\vdots$$
$$E_n \sqsubseteq E$$
$$E \sqsubseteq E_1 \sqcup E_2 \sqcup \ldots \sqcup E_n$$

These formulas are satisfied by $\mathcal{J}$ because $E^{\mathcal{I}} = E^{\mathcal{J}}$ for all entities.

Entities disjoint subsumption $\mathcal{I}$ satisfies the following condition:
For each entity $E, E_1, \ldots, E_n$, $\{E_1, \ldots, E_n\}$DISJ E implies $E_i^{\mathcal{I}} \subseteq E^{\mathcal{I}}, 1 \leq i \leq n$ and

$$E_k^{\mathcal{I}} \cap E_j^{\mathcal{I}} = \varnothing, k, j \in \{1, \ldots, n\}, k \neq j$$

The embedding rule **ER-D1** is applied and the following DL axioms are in $\Sigma_{\mathcal{ALCIN}}^{zero}$

$$E_1 \sqsubseteq E$$
$$E_2 \sqsubseteq E$$
$$\vdots$$
$$E_n \sqsubseteq E$$
$$E_i \sqsubseteq \prod_{j=i+1}^{n} \neg E_j, \text{ for } i = 1, \ldots, n-1 \tag{19}$$

The first n formulas are satisfied by $\mathcal{J}$ because $E^{\mathcal{I}} = E^{\mathcal{J}}$ for all entities.
Consider the DL axiom (19). Let $e \in E_i^{\mathcal{J}}$. As $E_k^{\mathcal{J}} \cap E_j^{\mathcal{J}} = \varnothing, k, j \in \{1, \ldots, n\}, k \neq j$, and $E^{\mathcal{I}} = E^{\mathcal{J}}$ for all entities, then $e \notin E_j^{\mathcal{J}}, j \in \{1, \ldots, n\}, i \neq j$. Thus $e \in \Delta^{\mathcal{J}} \backslash E_j^{\mathcal{J}}, j \in \{1, \ldots, n\}, i \neq j$. Therefore, $e \in (\prod_{j=1}^{i-1} \Delta^{\mathcal{J}} \backslash E_j^{\mathcal{J}}) \sqcap (\prod_{j=i+1}^{n} \Delta^{\mathcal{J}} \backslash E_j^{\mathcal{J}})$.

$$\therefore e \in \prod_{j=i+1}^{n} \neg E_j^{\mathcal{J}}$$

Hence, $\mathcal{J}$ is a model for $\Sigma_{\mathcal{ALCIN}}^{zero}$.

($\Leftarrow$) By the tree-model property we know that if E is satisfiable w.r.t. the $\mathcal{ALCIN}$ knowledge base $\Sigma^{zero}_{\mathcal{ALCIN}}$ then there exists a tree-like model $\mathcal{J} = (\Delta^{\mathcal{J}}, .^{\mathcal{J}})$ of $\Sigma^{zero}_{\mathcal{ALCIN}}$, such that $E^{\mathcal{J}} \neq \varnothing$. From such a tree-like model we can build an instantiation $\mathcal{I} = (\Delta^{\mathcal{I}}, .^{\mathcal{I}})$ of D^{zero}_{EER} such that $E^{\mathcal{I}} \neq \varnothing$, as follows:

$\Delta^{\mathcal{I}} = \bigcup_{\mathbf{E} \in \mathcal{E}} \mathbf{E}^{\mathcal{J}}$, where $\mathcal{E}$ denotes the set of all entities in D^{zero}_{EER}.

$E^{\mathcal{I}} = E^{\mathcal{J}}$ for all entities E in D^{zero}_{EER}

For binary relationship R, we define

$$R^{\mathcal{I}} = \{(e1, e2) | \exists t \in R^{\mathcal{J}}. \bigwedge_{i=1}^{2} (t, e_i) \in r^{\mathcal{J}}_{Ei}\}$$

For every role

Since $\mathcal{J}$ is a tree-like model, it is guaranteed that there is only one object t in an objectified relation $R^{\mathcal{J}}$ representing a given tuple in R. We must check that $\mathcal{I}$ is indeed an instantiation of D^{zero}_{EER} with $E^{\mathcal{I}} \neq \varnothing$.

Binary relationships $\mathcal{J}$ satisfies this $\mathcal{ALCIN}$ axioms

$$\exists r_{E1} \sqsubseteq R$$

$$\exists r^{-}_{E1}.\top \sqsubseteq E1$$

$$\exists r_{E2} \sqsubseteq R$$

$$\exists r^{-}_{E2} \sqsubseteq E2$$

$$R \sqsubseteq \ \exists r_{E1} \sqcap (\leq 1 \ r_{E1}) \sqcap \exists r_{E2} \sqcap (\leq 1 \ r_{E2})$$

that have been obtained by applying the KF embedding rule ER-A1.

By definition,

$$R^{\mathcal{I}} = \{(e1, e2) | \exists t \in R^{\mathcal{J}}. \bigwedge_{i=1}^{2} (t, e_i) \in r^{\mathcal{J}}_{Ei}\}$$

$$r^{\mathcal{J}}_{E1} = \{(t, e_1) | t \in R^{\mathcal{J}}\}$$

$$r^{\mathcal{J}}_{E2} = \{(t, e_2) | t \in R^{\mathcal{J}}\}$$

Therefore $\mathcal{I}$ satisfies the condition: For each relationship $R = \langle r_{E1} : E_1, r_{E2} : E_2 \rangle$, then

$$\forall (e_1, e_2) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}.(e_1, e_2) \in R^{\mathcal{I}} \rightarrow e_1 \in E_1 \wedge e_2 \in E_2$$

since for every tuple $(e1, e2)$ the $\mathcal{ALCIN}$ axioms above ensure the domain and range of R.

Cardinality constraint The following $\mathcal{ALCIN}$ axioms

$$E_1 \sqsubseteq \ (\geq min1 \ r^{-}_{E1}) \sqcap (\leq max1 \ r^{-}_{E1})$$

$$E_2 \sqsubseteq \ (\geq min2 \ r^{-}_{E2}) \sqcap (\leq max2 \ r^{-}_{E2})$$

are satisfied by $\mathcal{J}$.

Thus each $e \in E1^{\mathcal{J}}$ there exists $t_1, \ldots, t_m$, $min1 \leq m \leq max1$, such that $(t_i, e) \in r^{\mathcal{J}}_{E1}$.

By definition, of $R^{\mathcal{I}}$ we can ensure that the following condition:
For each relationship $R = \langle r_{E1} : E_1, r_{E2} : E_2 \rangle$, then

$$CARD_R(R, r_{E1}, E_1) = (min1, max1) \rightarrow \forall e_1 \in E_1^{\mathcal{I}}.min1 \leq |(e_1, e) \in R^{\mathcal{I}}| \leq max1$$

and

$$CARD_R(R, r_{E2}, E_2) = (min2, max2) \rightarrow \forall e_2 \in E_2^{\mathcal{I}}.min2 \leq |(e, e_2) \in R^{\mathcal{I}}| \leq max2$$

is satisfied by $\mathcal{I}$.

Subtype In this case, the following DL axiom is in $\Sigma^{zero}_{\mathcal{ALCIN}}$

$$E_2 \sqsubseteq E_1$$

E_1 and E_2 two entities and it is satified by $\mathcal{J}$. This encoding is derived from the KF embedding rule **ER-S1**. Thus a subsumption between the entities E_2 and E_1 is in D^{zero}_{EER}. As $\mathcal{J}$ is a model of $E_1^{\mathcal{J}} \sqsubseteq E_2^{\mathcal{J}}$ and for all entities, we have defined $E^{\mathcal{I}} = E^{\mathcal{J}}$. Therefore, the condition: E_1 ISA E_2 implies $E_1^{\mathcal{I}} \subseteq E_2^{\mathcal{I}}$ is satisfied by $\mathcal{I}$.

Subtyping of Relationship Let R_1 and R_2 two relationships. $\mathcal{J}$ satisfies the DL encoding

$$R_1 \sqsubseteq R_2$$

This formula is obtained from the KF embedding rule **ER-SA1**.
By definition of the interpretation $\mathcal{I}$ for relationship, it satisfies R_1 ISA R_2 implies $R_1^{\mathcal{I}} \subseteq R_2^{\mathcal{I}}$.

Mandatory relationship The DL encoding

$$E1 \sqsubseteq \ (\geq 1\ r^{-}_{E1})$$

expressing that r_{E1} is a role mandatory, is satisfied by $\mathcal{J}$. Thus for every element in the entity E_1, it must be the case that is in relation R.
Thus the condition, for each relationship $R = \langle r_1 : E_1, r_2 : E_2 \rangle$, then

$$CARD_R(R, r_1, E_1) = (1, max1) \rightarrow \forall e_1 \in E_1^{\mathcal{I}}.1 \leq |(e_1, e) \in R^{\mathcal{I}}| \leq max1$$

is satisfied by $\mathcal{I}$.

Entities completeness constraints Subsumption The following DL axioms in $\Sigma^{zero}_{\mathcal{ALCIN}}$

$$E_1 \sqsubseteq E$$
$$E_2 \sqsubseteq E$$
$$\vdots$$
$$E_n \sqsubseteq E$$
$$E \sqsubseteq E_1 \sqcup E_2 \sqcup ... \sqcup E_n$$

are satisfied by $\mathcal{J}$. The embedding rule that has been applied is **ER-C1**.
Since $E^{\mathcal{I}} = E^{\mathcal{J}}$ for all entities, then the following conditions:
For each entity $E, E_1, \ldots, E_n$, $\{E_1, \ldots, E_n\}$COV E implies $E_i^{\mathcal{I}} \subseteq E^{\mathcal{I}}, 1 \leq i \leq n$ and $E^{\mathcal{I}} \subseteq \bigcup_{i=1}^{n} E_i^{\mathcal{I}}$
is satisfied by $\mathcal{I}$.

Entities disjoint subsumption $\mathcal{J}$ is a model of the following DL axioms in $\Sigma^{zero}_{\mathcal{ALCIN}}$

$$E_1 \sqsubseteq E$$
$$E_2 \sqsubseteq E$$
$$\vdots$$
$$E_n \sqsubseteq E$$

$$E_i \sqsubseteq \prod_{j=i+1}^{n} \neg E_j, \text{ for } i = 1, \ldots, n-1$$

They were obtained appling the embedding rule **ER-D1** from the following condition:
For each entity $E, E_1, \ldots, E_n$, $\{E_1, \ldots, E_n\}$DISJ E implies $E_i^{\mathcal{I}} \subseteq E^{\mathcal{I}}, 1 \leq i \leq n$ and $E_k^{\mathcal{I}} \cap E_j^{\mathcal{I}} = \varnothing, k, j \in \{1, \ldots, n\}, k \neq j$
The first n conditions are satisfied by $\mathcal{I}$ because $E^{\mathcal{I}} = E^{\mathcal{J}}$ for all entities.
Consider the last condition. Let suppose that $e \in E_i^{\mathcal{I}} \cap E_k^{\mathcal{I}}, i \neq k$. If $i < k$ then

$$E_i \sqsubseteq \prod_{j=i+1}^{n} \neg E_j$$

$$E_i \sqsubseteq \neg E_{i+1} \sqcap \ldots \sqcap \neg E_k \sqcap \ldots \sqcap \neg E_n \tag{20}$$

As $e \in E_k^{\mathcal{I}}$ then $e \in E_k^{\mathcal{J}}$ and $e \notin \neg E_k^{\mathcal{J}}$ and therefore $e \notin \prod_{j=i+1}^{n} \neg E_j^{\mathcal{J}}$. This contradicts the DL axiom refdisj-ER-vuelta.
Similarly, is the case when $i > k$ considering

$$E_k \sqsubseteq \prod_{j=k+1}^{n} \neg E_j$$

Thus $e \notin E_i^{\mathcal{I}} \cap E_k^{\mathcal{I}}, \ \forall i, k \in \{1, \ldots, n\}, i \neq k$.
$\therefore E_i^{\mathcal{I}} \cap E_k^{\mathcal{I}} = \varnothing, \ \forall i, k \in \{1, \ldots, n\}, i \neq k$.

Hence, $\mathcal{I}$ is a model for D^{zero}_{EER}.

□

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